The Role of Expected Surplus in Online Auctions

Viet Vu, and Logan Graham

January 21, 2016

Abstract

In this paper, we look at the effect of a strategy employing discounting, cross-bidding, and expectations of auction outcomes on bidder and seller surplus in online auctions. First, we extend Peters and Severinov (2006) and develop a theory that predicts an equilibrium which increases system surplus through using this "enhanced" strategy. Then, we use real life eBay data to create a computer program which simulates an online auction market place where we are able to test the effect of this strategy. We find that total system surplus can be increased by bidders using the enhanced strategy, notably by continually updating their beliefs on auction outcomes. These findings imply a more efficient and profitable mechanism that can be used by online auction marketplaces.

JEL Classifications: D44

1 Introduction

In standard auction theory, the typical mechanism of analysis is a single auction or multiple concurrent auctions beginning and ending at the same time. As a result, entry and exit dynamics and their effects on bidder behaviour have not been significantly studied. And in fact, those bidding behaviours can be quite different from what is normally described. For example, bidders participating in an eBay auction follow a variety of strategies, from last-minute "sniping" to bidding only a fraction of their value in many auctions in the hopes that they will win one.

Approaches to auction theory can be generally classified in two distinct approaches - an epirical approach and a theoretical approach. The empirical approach looks at empirical auction results to draw inferences and conclusions regarding bidder behaviour. It is mostly used to understand bidding dynamics within an auction system and can be used to offer predictions for some markets. This approach has yielded relative success in the field. They examine cross-bidding behaviour as in Anwar et al. (2006), sniping-behaviour as in Roth and Ockenfels (2002) as well as finding the relationship between final prices and factors such as length of auctions, seller feedback and starting(reserve) prices Lucking-Reiley et al. (2007). Some researchers have gone as far as to build auction simulations that tries to predict auction outcome from available information such as Wang et al. (2008), Mizuta and Steiglits (2000)

On the theoretical side, many theories were developed to find areas where strategic interaction between bidders and sellers become significant. On the bidder side, specific observed strategies are analyzed such as the sniping strategy in Hendricks et al. (2012) and Mizuta and Steiglits (2000) or endogeneous entry decisions as in Levin and Smith (1994). On the seller side, decisions regarding setting the reserve price in Jehiel and Lamy (2014), starting prices in Tan et al. (2010) or even the seller experimentation process as in Einav et al. (2012)

Peters and Severinov (2006) is a theoretical approach to analyze online auctions. They identified an equilibrium strategy when auctions end. The strategy consists of a bidder bidding on the auction with the minimum standing bid below their value, when he or she enters the market or is outbid by another bidder. The auction ends when all bidders have entered the market, and when bidders cannot or do not want to bid anymore. Anwar et al. (2006) tested the implications of this theory empirically by identifying a cluster of auctions finishing at similar times and found that bidders who cross-bid between multiple auctions finishing closely together paid 91% of the price paid by bidders who did not cross-bid.

This paper extends the ideas presented in Peters and Severinov (2006) further by looking at how bidders bid when they perceive the auction market as a continuous series of bidders and sellers entering and exiting the market. The advantage of this is that it models a better representations of real life conditions a typical bidder would face in online auctions.

There are two major differences between our environment and the environment described by Peters and Severinov. First, because auctions enter and exit at different times, in order to evaluate all auctions at a given time, bidders form beliefs about the future value of auctions. Secondly, as the auction market considered by Peters and Severinov ended at the exact same time, discounting between auctions were not significant in bidders' strategic decisions. However, empirical results seem to suggest that some bidders do discount future auctions in a second-price setting Deck and Jahedi (2014).

The difference in environment necessitates bidders to form expectations on the finishing prices of auctions. We argue that bidders do this by ranking the auctions in terms of their expected surplus and bids on auctions that will give them the most surplus at a given time. As they can switch auctions at any given time, they do not run the risk of being locked in to an auction that ends up giving them a lower surplus.

In order to test our theory, we construct an auction simulation that tests two distinct scenarios. A first scenario, where bidders look over all the current auctions, chooses an auction with the lowest standing bid below their value and restrict their bids in that auction. The second scenario is where bidders cross-bid according to the strategy we devise, where they form a belief on the closing price of an auction, and bid on the auction that gives them the highest surplus.

We find that the strategy we devise increases overall system surplus, compared to the first strategy. The increase is due to a higher probability of trade happening (an auction has at least one bidder bid on it), as well as improved matching of high valued bidders to sellers. These findings suggest that if an e-commerce marketplace were to introduce a mechanism to enable or incentivize this strategy to take place, there would be practical real-life benefits for the bidders, the sellers, and the company as well.

Section 2 describes the the data we use. Section 3 describes the theoretical justification for discounting and updating expectations. Section 4 describes our simulation. Section 5 discusses our results and section 6 concludes.

2 Data

2.1 Motivation

In order to create an accurate and useful simulation, data on over a month of real-life eBay auctions are collected. This serves three purposes. First, we use this data to discover general bidder and seller characteristics so that we can create a basic simulation of an online auction. Second, we use it to test the accuracy of our simulation given some well-defined parameters. Last, we use this data to establish baseline price, surplus, and distribution metrics to analyze the impact of changes in decision rules.

2.2 Auction Mechanism in Data

Anwar et al. (2006) collected data on 105 auctions from the online ecommerce site, eBay. An eBay auction is second-price auction, which awards the good to the highest bidder at the second highest bid.

From the bidder perspective, a bidder makes three choices: if and when to bid, in which auction to bid, and how much to bid – the latter two choices invoking a strategy we study in this paper.

When a bidder is choosing which auction to bid in, they have access to information from a set of currently ongoing auctions, each displaying their duration and current standing bid. For each auction, the current standing bid is the second highest bid in the auction. After reviewing this information, the bidder can engage in the auction by submitting a bid that is above the current standing bid, typically by a standard increment (which is dependent on the current standing bid). If the bidder's bid is higher than the old highest bid (which is not seen by an individual bidder), then the old highest bid becomes the current standing bid and the bidder's bid becomes the new unseen highest bid. Conversely, if the bidder's bid is lower than the highest bid, the current standing bid will be updated to the new bidder's bid.

For example, an auction has 2 bidders. Bidder 1 inputs \$150 as her maximum bid. Bidder 2 inputs \$80 as her maximum bid. When Bidder 3 enters the market, she sees the current standing bid as \$80. Bidder 3 then submits a new bid and inputs \$120. Because this is smaller than \$150, but higher than Bidder 2's bid, the current standing bid updates to \$120. Bidder 3 then decides to bid again and inputs \$160. This time, her bid is successful and Bidder 3 knows this as the standing bid is updated to \$150, which is below her input.

An interesting component of eBay auctions is that bidders are given the opportunity to employ a proxy-bidding mechanism. In the proxy-bidding mechanism, a bidder inputs their maximum willingness to pay in an auction. The proxy-bidding mechanism then monitors the auction continuously and if the bidder is outbid, the mechanism automatically places a higher bid for the bidder by a small increment. When the bid reaches the maximum bid, the system notifies the bidder an asks if the bidder wants to update their bid or not. This is useful for our analysis because the highest bid recorded by the system for each user that lost an auction represents their maximum willingness to pay for that good.

From the seller perspective, a seller makes three choices: when to start an auction, how long the auction should last, and how to set the starting price. The institutional framework under which our data was collected dictated that auctions could last either 3, 5, or 7 days, after which the auction would terminate. Therefore, "sniping", or last-minute bidding, is a viable strategy, and one shown to potentially be advantageous to a bidder (Roth and Ockenfels, 2002); however, this is out of the scope of this paper. The last choice a seller makes – where to set the starting price – is interesting due to incentives we describe later in this section.

2.3 Overview of collected data

Anwar et al. (2006) developed a web crawler which collected data on 105 eBay auctions between May 28th 2001 and July 3rd 2001 for a new Pentium III 800Mhz processor. This item was chosen as different units of the processors were virtually identical. Furthermore, we believe the effect of enhanced listings (with pictures) is minimized by the homogeneous nature of the product.

2.4 Bidders

Analyzing unique bidders in the data served to establish a belief about the distribution of valuations. This is important for two reasons: first, bidder valuations are key components in a bidder's bidding strategy; second, they allow us to analyze the impact on surplus of changes to decision strategies. We identified 533 unique bidders in the data. Table 1 provides the description of characteristics we analyzed for each bidder.

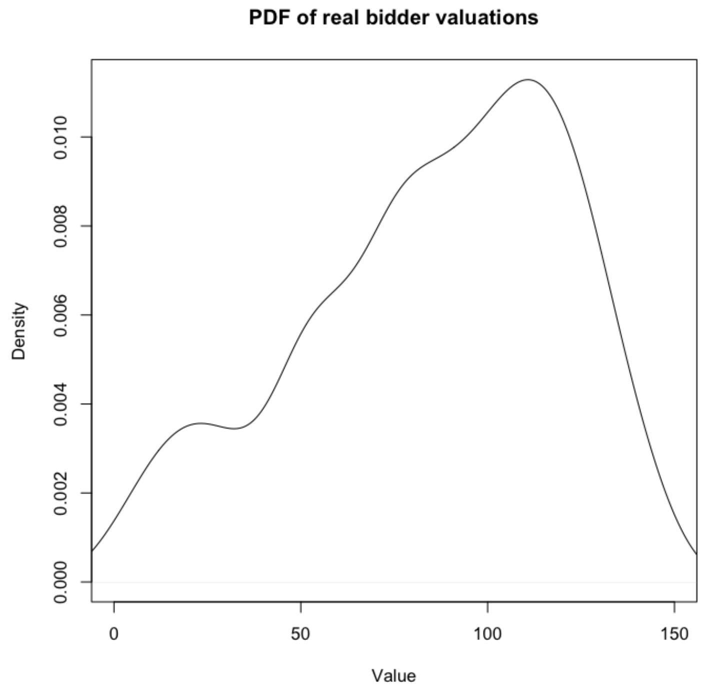
Variable	Description
auction_number	ID of the auction
bidder	Username of the bidder
bidder_feedback	Number of Positive feedback received by bidder
bid_amount	Recorded bid submitted by the bidder
bid_time	Timestamp of the recorded bid

Table 1: Variable description for bidders.

In constructing an accurate simulation we identified a series of key metrics.

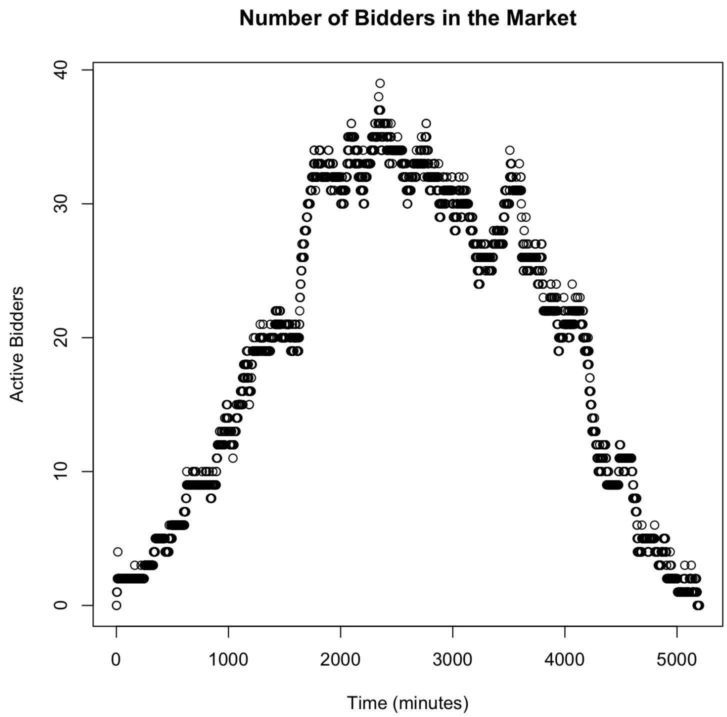
We identified a bidder's highest bid (especially for those who lost auctions) as the best approximation of a bidder's valuation. At best, this number is a bidder's valuation, and at worst it is the lower bound on a bidder's valuation. This is because the bid's interpretation depend on whether or not that bidder has won the auction. For a bidder who lost an auction, by virtue of the eBay framework, their highest bid is their value: they are not willing to bid any higher. However, if a bidder has won, one cannot be certain that the bid which they submitted is their valuation.

We noticed several interesting characteristics of the timing dynamics of bidders. We measured entry as the date at which a bidder placed their first bid, and their total duration as the point at which they place their last bid minus their entry point. Our analysis revealed that the probability a bidder would enter at a given point is roughly uniform, with spikes



occurring (expectedly) at the start of a new day. The reason for the initial build up of active bidders at the very beginning and the decline towards the very end, is due to the nature of the collected data. The crawler only kept track of certain auctions; after a certain date, the web crawler no longer tracked new auctions. Therefore, as the number of auctions declined, so too did the number of active bidders. Realistically, we would expect the number of active bidders to remain roughly constant over a given time interval.

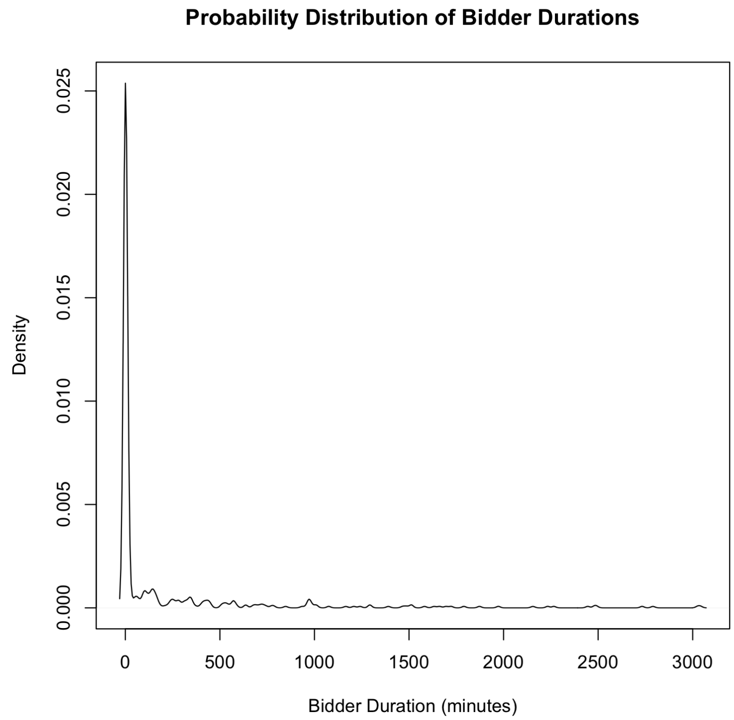
The amount of time a bidder stayed was, on average, extraordinarily small. It is difficult to conceive how an auction could be efficient if bidders only stay to make a short series of bids before disappearing. We reconcile this by acknowledging that we cannot know how long a bidder stays in the market before and after their bids or how long they were "lurking". In order to randomize the entry choice, and to account for lurking, our model randomize entry time as well as duration.



Further analysis revealed that of the 533 bidders, 21.39% placed bids in multiple auctions at the same time (”cross-bidding”). In addition, 17.6% of bidders who had won an auction placed further bids after they had won, indicating preference for more than one good. The number of bidders active in the market at any one point reached a maximum of 39.

2.5 Sellers

Sellers were represented by 105 individual auctions during the relevant period. Each auction lasted 3, 5, or 7 days, with the length equally distributed between the three values. Of these auctions, we only find 63 unique usernames for sellers, implying that some sellers sold more than one item of the good. To create our auction simulation, we analyze the timing and pricing dynamics in our sample. Table 2 describes the variables we look at for

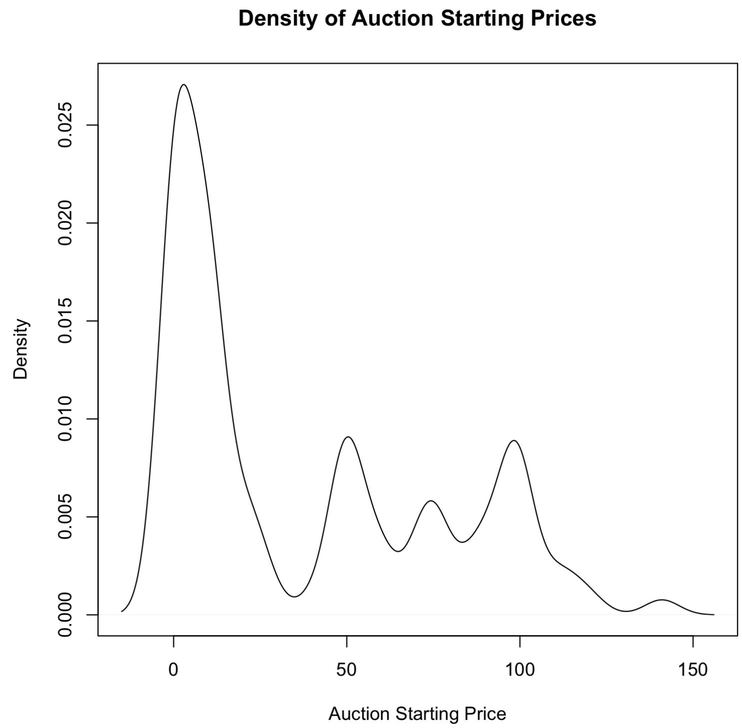


each seller.

	Mean	Std. Deviation
Final Price	116.29	15.64
Starting Price	38.47	39.76
Number of Bids	13.02	7.37
Days	5.16	2.05

Table 2: Linear regression results for determinants of final prices.

Interestingly, the distribution of auctions' starting prices was abnormal. There was a cluster around \$0 for starting prices. In fact, 42.8% of the auctions had a starting price of lower than \$10. We motivated this as a seller strategy to engage more bidders in their auction in the hopes of increasing competition and ultimately their final selling price. EBay also charges sellers a fee proportional to their starting price, creating an incentive to set a lower starting price, especially when sellers are confident they'll receive a competitive price.



To explore these effects further and to establish outcome expectations for our later analysis we analyzed the impact of auction variables on the final prices.

$$final\_price = \beta_0 + \beta_1 starting\_price + \beta_2 number\_of\_bid + \beta_3 days$$

An Ordinary Least Squares regression identified a strong relationship between the starting price, bidder competition, and the length of the auction, with the final price. Although the R^2 is relatively low, longer auctions are correlated with a higher closing price; auctions with more bids are correlated with a higher closing price, and auctions with a higher starting price had a higher closing price. Table 3 summarizes our results.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	89.1417	6.3686	14.00(***)	0.0000
Starting Price	0.1416	0.0529	2.68(**)	0.0087
Number Of Bids	0.8722	0.2938	2.97(**)	0.0037
Duration	2.0033	0.7536	2.66(**)	0.0091
R^2	0.172			
n	105			

Table 3: Linear regression results for determinants of final prices.

3 Model

3.1 Environment

The auction mechanism used in this model is an incremental-bid second-price auction mechanism. There are m auctions and n bidders that enter and exit the market randomly. Each bidder demands one unit of the good. The good sold in the market is homogenous. Each bidder has a private value from a known distribution.

Bidders also hold a belief regarding the probability that they will win an auction. This probability of winning is a function of their private value and is called $Q(v)$. This valuation is drawn from a publicly known value distribution. Bidders also know when an auction will end and the current highest bid in any current auctions.

Bidding happens sequentially with each bidder acting individually at a distinct moment from other bidders. Any new bid is a small fixed increment from the current bid, conditional on the current bid (If the auction is $1 or less, the increment is $0.05; if it is between $1 and $5, the increment is $0.25 and so on.) For simplicity, we generalize this incremental amount to a fixed level, irrespective of the current standing bid. This models the proxy bidding mechanism used by Ebay. One concern with using sequential bidding is that the number of rounds where each bidders have a chance to bid affect the final outcome. We tested a variety of different cases (where each turn is defined as a minute, five minutes, ten minutes, and finally half an hour.) No significant deviation in outcome took place.

The good is awarded to the highest bidder at the time the auction closes, at the second highest bid. The market closes when all available auctions have entered the market and announced a winner (or lack thereof.)

3.2 Outcome

For the first part of this section, we will reconstruct the bidding strategy as defined in Peters (2006.) We will then extend this result to show how bidders choose amongst different auctions.

We can easily calculate the bidder's expected payoff of an auction. It is the difference between their valuation multiplied by the probability trade happens $Q(v)$ and the expected price they pay for that auction:

$$Q(v)v - P(v)$$

In the case of a Bayesian incentive compatible equilibrium, using this strategy must at least give the bidder the same surplus as if they were using another strategy; this means that for all $v' \neq v$:

$$Q(v)v - P(v) \geq Q(v')v - P(v')$$

Therefore, at $v' = v$, the bidder must be maximizing; in other words:

$$Q'(v')v - P'(v') = 0$$

$$=> Q'(v')v = P'(v')$$

$$=> \int_{v_{min}}^{v} Q'(v')v'dv' = \int_{v_{min}}^{v} P'(v')dv'$$

Integrating the left-hand side by parts gives:

$$Q'(v)v - Q'(v_{min})v_{min} - \int_{v_{min}}^{v} Q(v')dv' = P(v) - P(v_{min})$$

If we allow $Q(v_{min}) = P(v_{min}) = 0$, by reordering the expressions, we have:

$$\int_{v_{min}}^{v} Q(v')dv' = Q(v)v - P(v)$$

This implies that agents can calculate their predicted surplus by looking at the integration up to their valuation of the probability they win an auction. Let's call this value $E(v)$. When a cross-bidder looks at a group of auctions, she constructs the following index for every auctions:

Definition 1.1: The **Sergei index** si_n^m for an auction n by a bidder m is the difference between the surplus gained by bidder m and the bidder's predicted surplus $E(v^m)$ discounted by time:

$$si_n^m = (v^m - sb_n - E(v^m))\sigma^{ln(t_n)}$$

$$E(v^m) = \int_{sb}^{v} Q(v')dv'$$

where v^m is the bidder's valuation, sb_n is the auction's current standing bid, and $\sigma^{ln(t_n)}$is the discount factor to the power of the natural log of the time remaining in the auction. The current standing bid of the auction is taken as the lower bound as bidders who have the valuation lower than the standing bid has no chance of winning the auction and thus v_{min} for the set of bidders bidding in that auction is the standing bid of that auction.

The natural log is taken of the time that remains in an auction in order to scale the time across auctions ending at drastically different timing. We now defines the sergei strategy using Definition 1.1:

Definition 1.2: **Sergei strategy** is a strategy followed by bidders in an auction. It is characterized by the following steps:

1. If the bidder has a highest bid in an auction, the bidder doesn't act.
2. If the bidder doesn't have a highest bid in any auctions, the bidder constructs a sergei index for all avialble auctions.
3. The bidder finds the auction with the highest non-negative sergei index and bids the minimum increment allowed in that auction.
4. If all sergei indexes are non-positive, the bidder does not act.

Intuitively, bidders attach a price prediction for each active auction in the market. They then choose the auction that will give them the highest level of surplus. This implies that the expected surplus for the bidder following this strategy is:

$$(1 - P(sp < E(v))^n)E(sp|sp > E(v))$$

Where n is the number of auctions that the bidder has bid in, and sp is the bidder surplus, defined as the difference between the bidder's private value and the price they end up paying in the end. This implies that with a positive standard deviation in the distribution of surplus, as $n \to \infty$, $P(surplus > E(v)) \to 1$. Intuitively, as the number of auction increases, the chances that the bidder find an auction that gives them a higher surplus increases. This also allows us to calculate the probability of winning any auction above the expected value given the number of auctions n and the probability of doing worse than expected surplus in an auction.

Lemma 1.3: Given any surplus distribution, there exists a number of auction n that makes the expected surplus gained using Sergei strategy at least as good as E(v) calculated as in definition 1.1 . In other words:

$$\forall E(v), \exists\, n\ s.t. (1 - P(sp < E(v))^n) E(sp|sp > E(v)) \geq E(v)$$

Proof of lemma 1.3:

First, we observe that in a second-price auction context, it is never optimal for bidders to bid above their valuation. Therefore, $sp \geq 0 \forall v > 0$. We also observe, that this surplus is distributed over a distribution D conditional on the bidders value: $sp \sim D(v)$. By virtue of how expected surplus is calculated, the probability that a given surplus is below this value is smaller than 1: $P(sp < E(v)) < 1$. Therefore, as $n \to \infty$, $P(sp < E(v))^n \to 0$.

For each surplus distribution, we can also construct an expected value conditional on that value exceeding the expected surplus; that is $E(sp|sp > E(v))$. This number will be greater than or equal to $E(v)$.

The probability that a bidder does better than expected surplus in at least one auction is $1 - P(sp < E(v))^n$. This means that the expected surplus by using sergei strategy becomes:

$$E(sp|sp > E(v))(1 - P(sp < E(v))^n)$$

As $E(sp|sp > E(v)) \geq E(v)$, $\exists 1 \geq \alpha > 0$ s.t. $\alpha E(sp|sp > E(v)) = E(v)$. Therefore, we can find an n such that $1 - P(sp < E(v))^n \leq \alpha$. Hence, for all surplus distributions, we can always find an n such that:

$$E(sp|sp > E(v))(1 - P(sp < E(v))^n) \geq E(v) \blacksquare$$

This result also leads us to the following theorem that characterizes an equilibrium condition for the auction market:

Theorem: Sergei strategy is a better strategy to follow than the non-Sergei strategy..

Proof of theorem: We start by specifying the functions that are defined above. First, we let s_{min}, s_{max} be the bound on the surplus. For a surplus s: $0 < s_{min} \leq s \leq s_{max}$, we have:

$$P(sp < s) = g(s) = \int_{s_{min}}^{s} p(s')ds'$$

$$E(sp|sp > s) = h(s) = (1 - g(s)) \int_{s}^{s_{max}} s'p(s')ds'$$

Where $p(s')$ is the probability that a bidder will receive a surplus of value s'. We have the bidder's expected surplus as:

$$h(s)(1 - g(s)^n)$$

It's corresponding derivative, with respect to the chosen level of s is:

$$h'(s) - g(s)^n h'(s) - nh(s)g'(s)g(s)^{n-1} = 0[1]$$

Where s is the cut-off surplus value that the bidder chooses to make their calculation. If a player is not playing the Sergei strategy, they look at the available auction and choose the auction with the lowest standing bid. This means that, given lack of information, they randomly choose a surplus from the surplus distribution. Their expected surplus is therefore $E(v)$. We now need to show that this equation holds for when $s = E(v)$.

We look at the case of when surplus is uniformly distributed from 0 to a value $m > 0$. In that case:

$$g(E(v)) = 0.5$$
$$h(E(v)) = \tfrac{3m}{4}$$
$$h'(E(v)) = 0.5$$

By simplifying [1], we get the following equality:

$$1 + 3n = 2^n$$

Solving this for n yields n=3.54. We can interpret this value as the number of auctions that the bidders need to see for choosing expected value to be the best reply by that bidder in a uniform distribution case. This gives the bidder an expected payoff of 0.68m, higher than the expected payoff of 0.5m the bidder would have gotten had she pursued any positive surplus. This also implies that if the bidder sees more auction, the best reply cut-off surplus will also be higher.

In the case of bidders who don't cross-bid, a slight modification is required. As these bidders do not anticipate to bid in more than one (sometimes two) auctions, the probability of them getting a surplus above their expected surplus is low. Therefore, these bidders will bid the auctions they are in up to their value in order to maximize the chance of winning any surplus.

We now explore how our simulation constructs the environment as specified above and incorporates the strategies identified.

4 Simulation

4.1 Overview

In order to test the effect of different bidding strategies, we construct a simulation of an n-period online auction. In order to validate our findings, we generate the starting conditions such that it is similar to the collected data.

4.2 Bidders description

In our simulation, bidders have three significant characteristics: valuation, duration in market, and their strategy type.

Firstly, when the simulation generates a bidder, it draws an arbitrary valuation from a distribution which emulates the distribution found in data. Specifically, we draw from the distribution of bidder valuations described in the previous section.

Secondly, the simulation draws a time-period entry point randomly from a uniform distribution. Then, it draws a time-period where bidders stop considering new auctions from a uniform distribution between one and five days after the entry point. This duration is significantly longer than the mean duration of bidders in the data. We justify this by appealing to the lurking principles as explained in the previous section.

It is also important to stress that bidders do not exit the market immediately after their end time. They simply stop considering new auctions to bid in. This exit condition allows bidders to finish auctions that they participated in. The bidder formally exits when all auctions they participated in closes.

Lastly, the simulation draws whether or not not the bidder is a Sergei-type bidder. A bidder can either be a Sergei-type bidder or a non-Sergei-type bidder. A Sergei-type bidder cross-bid, construct an index for each auction, and compare the expected surplus of that auction to their overall expected surplus. A non-Sergei-type bidder picks the auction with the lowest current standing bid unless the original auction ends (and that they're still in the market.) Sergei players construct their index using the following rule:

Definition 1.1: The **Sergei index** si_n^m for an auction n by a bidder m is the difference between the surplus gained by bidder m and the bidder's predicted surplus $E(v^m)$ discounted by time where $Q(v)$ the probability they win an auction with value v; sb_n is the starting bid of an auction n, v^m is the value of bidder m, σ is the discount factor:

$$si_n^m = (v^m - sb_n - E(v^m))\sigma^{ln(t_n)}$$

$$E(v^m) = \int_{sb}^{v} Q(v')dv'$$

Before the marketplace is initiated, the proportion of bidders who are Sergei-type bidders is specified. This variation allows us to see the effect of bidding strategies on system surplus.

4.3 Sellers description

Extracting seller's valuation was more difficult than estimating the value for bidders. As eBay charges fees conditional on starting prices, there is a strategic decision involved in the starting price choice. Therefore, a normal distribution of seller value of the good is assumed with some sellers on the lower end of the distribution reporting their "type" to be lower.

There are two rationales for this approach. Firstly, as commented on in the previous section, auctions with lower starting price attract more bidders. In the case of bidders not cross-bidding across auctions, attracting more bidders can be a valuable way for sellers to increase their prices. Secondly, the lower the valuation of the seller, the less cost they assume by setting a low starting price. Therefore, sellers with lower valuations are more likely to have a low starting price. The difference between the distribution of simulated starting prices and the real starting prices are demonstrated in Figure 1

A two sample Kolmogorov-Smirnov conducted on the distribution from the data and the distribution generated using our hypotehsis gives the D-value of 0.1594 and a p-value of 0.1371. This implies the null hypothesis of equality of distribution cannot be rejected at the 10% level.

Other seller attributes were simpler to produce as the starting time and ending time of the auctions are present in the data and is drawn from a uniform distribution, similar to the case for bidders.

4.4 Simulation Mechanism

Before the simulation begins, the controller selects two parameters. Firstly, one selects how many auction iterations occur. Secondly, the controller determines the proportion

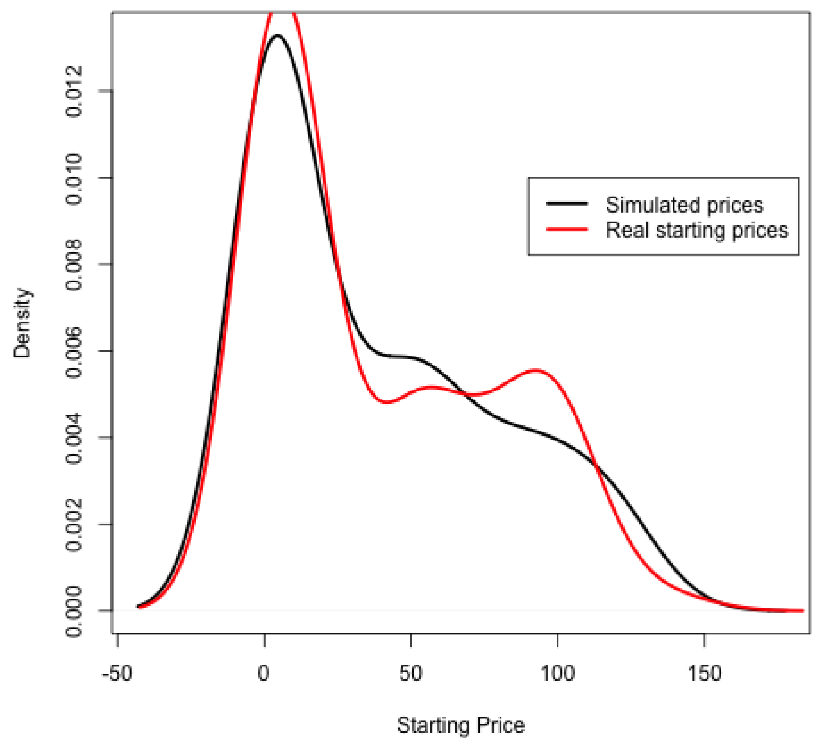


Figure 1: Probability density of starting prices

of bidders who are Sergei-type bidders. The first parameter is a tradeoff between output sample size and time; the more iterations, the longer the simulation runs, but the more data points one receives in the end. The second parameter is the variable of interest and is useful for testing surplus differences.

Sequentially, the simulation begins by creating two populations of agents: bidders and sellers according to the parameters described above. Then, the auction checks through every potentially active bidder. If the bidder is currently winning an auction, or if there are no auctions which give the bidder surplus above their expected surplus (following their strategy), then the bidder does not bid. After the simulation has checked every bidder for a potential action, the auction will check for auctions that have finished. If an auction finishes, the simulation labels a winner and both the auction and bidders are taken out of the populations of potential agents. This completes one round, which can be interpreted as the progression of time.

After every iteration, a list of bidder valuations and the surplus they received from an auction is added to a common data set. At the beginning of each iteration, bidders use this continually-growing data bank to calculate their expected surplus for their given valuation if they are Sergei bidders.

5 Results

Two simulations were conducted. The first simulation was ran with none of the bidders cross-bidding and in the second case, all bidders bid freely amongst auctions. The discount rate was kept constant in both of the simulations. The number of iteration between each sergei belief value update was 40 auctions. The simulation stopped when the updated value was within 3% of the original value. The summary of the result is presented in the table below:

Variable	Outcome Sergei	Outcome Non Sergei
Total System Surplus	\$5853.13	\$5671.83
Average Consumer Surplus	\$14.67	\$14.04
Average Seller Surplus	\$43.20	\$45.27
Average Price	\$111.38	\$111.51
Price Variance	\$30.74	\$127.01
Averaged number of closed auctions	101.1	95.63

Table 4: Simulation results

5.1 Findings

In general, following sergei strategy increases average per consumer surplus by 4% (or by $0.6 .) At the same time, it reduces seller surplus by 4.4% as well. However, there is a net increase of system surplus of 3% from the non Sergei case. This can be attributed to 5.7% more auctions closing as the result of following the sergei strategy. This implies that following the sergei strategy increase the probability of trade happening.

In addition, cross-bidding reduces the variance in closing prices. This indicates that cross-bidding creates a situation where better final price projection can be constructed. Increase in average consumer surplus can be attributed to better matchings produced by the strategy followed by bidders - it is less likely to create situations where high value bidders bid against each other.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.8692	0.0432	-20.12	2e-16
I(value^6)	2.171e-12	1.748e-14	124.15	2e-16

Table 5: Surplus function estimate - non Sergei

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.333	0.0345	-38.60	2e-16
value^6	2.634e-12	1.403e-14	187.8	2e-16

Table 6: Surplus function estimate - Sergei

These two estimation intersect at value equals to 100.065 where projected surplus is at 1.3. This means that up until value point 100, sergei and non-sergei strategy give bidders the same amount of surplus. However, there is a distinct advantage to bidding according to sergei strategy beyond the 100 value point.

However, looking at the bidder entry and exit behaviour, a peculiar point should be noted. In our simulation, it is assumed that the only chance for bidders to obtain the good

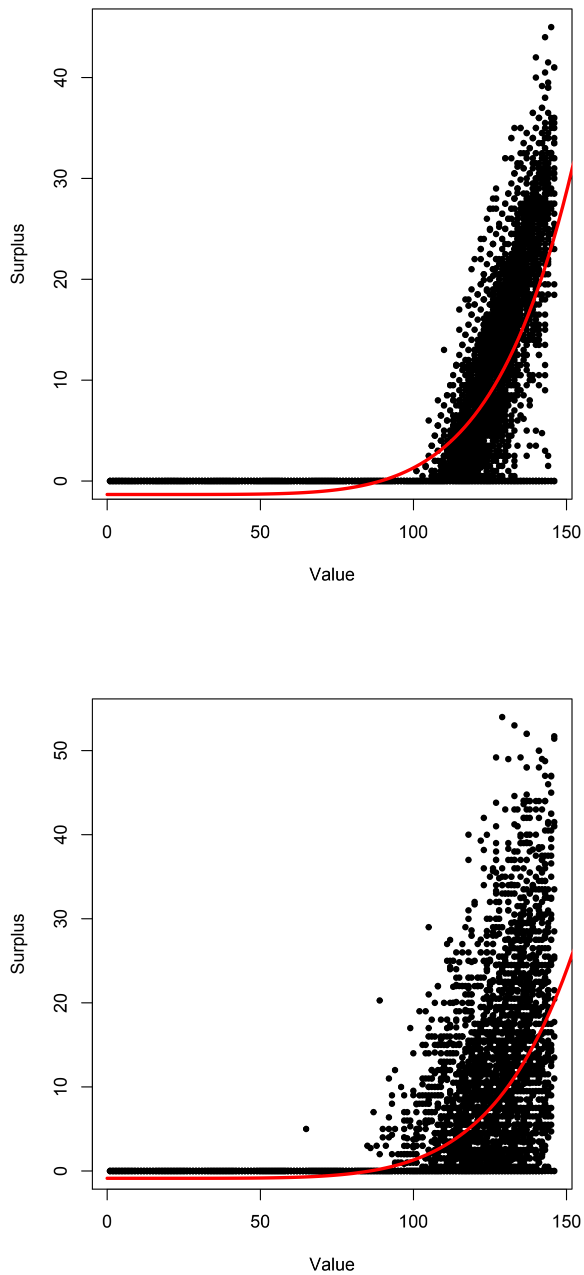


Figure 2: Value function estimates (top: Sergei ; bottom: Non Sergei)

is during the auction period. In real life, this is not the case as there is no hard "closing time" for auctions. It is much more likely that the simulation go on indefinitely with more auctions entering the market. This also means that there is a possible congestion problem where too few bidders are competing for too little goods within our simulation towards the beginning and the end.

In order to work around this issue, additional simulations were ran by assuming the auction market continue to operate for twice the duration. The numbers of bidders and sellers were increased accordingly while keeping their entry and exit rate relatively constant. (Un)surprisingly, this resulted in an almost identical outcome for average per bidder bidder surplus leading us to believe that the result is significant even in cases of auction markets happening indefinitely.

6 Conclusion

A few comments regarding our findings in in order. First of all, it is clear that under our system, total surplus experienced by buyers and sellers go up from the increased trading probability. However, the computation required to reach this equilibrium is quite challenging when bidders have to calculate a new set of index every round to update their expectations with the new information.

This suggests that if eBay could build a structure that allows the proxy bidding mechanism to bid across multiple auctions (by a user constructed list of identical items) to increase surplus in the segment of the market dealing with homogenous goods. Future research can examine the effect of this cross-bidding effect when the bidder is bidding on heterogenous goods (clothing) or examine cases when bidders demand more than one unit of a good (zippo lighters).

References

Anwar, Sajid, Robert MacMillan, and Mingli Zheng (2006), "Bidding behavior in competing auctions: Evidence from ebay." *European Economic Review*, 50, 317–322.

Deck, Cary and Salar Jahedi (2014), "People do not discount heavily in strategic settings but they believe others do." *NBER Working Paper*.

Einav, Liran, Theresa Kuchler, Jonathan Levin, and Neel Sundaresan (2012), "Learning from seller experiments in online markets." *Working Paper*.

Hendricks, Kenneth, Ilke Onur, and Thomas Wiseman (2012), "Last-minute bidding in sequential auctions with unobserved, stochastic entry." *Review of Industrial Organization*, 40, 1–19.

Jehiel, Philippe and Laurent Lamy (2014), "On absolute auctions and secret reserve prices." *CEPR Discussion Paper*.

Levin, Dan and James L. Smith (1994), "Equilibrium in auctions with entry." *American Economics Review*, 84, 585–599.

Lucking-Reiley, David, Dough Bryan, Naghi Prasad, and Daniel Reeves (2007), "Pennies from ebay:the determinants of price in online auctions." *Journal of Industrial Economics*, 55, 223–233.

Mizuta, Hideyuki and Ken Steiglits (2000), "Agent-based simulation of dynamic online auctions." *Simulation Conference Proceedings*.

Peters, Michael and Sergei Severinov (2006), "Internet auctions with many traders." *Journal of Economic Theory*, 130, 220–245.

Roth, Alvin E and Axel Ockenfels (2002), "Last minute bidding and the rules for ending second-price auctions: Evidence from ebay and amazon auctions on the internet." *American Economic Review*, 92, 1093–1103.

Tan, Chuan-Hoo, Hock-Hai Teo, and Heng Xu (2010), "Online auction: The effects of transaction probability and listing price on a sellers decision-making behavior." *Electron Markets*, 67–79.

Wang, Shanshan, Wolfgang Jank, and Falit Shmueli (2008), "Explaining and forecasting online auction prices and their dynamics using functional data analysis." *Journal of Business and Economic Statistics*, 26, 144–160.