LEARNING THROUGH SUCCESSES: REPUTATION EFFECTS IN A SOCIAL NETWORK

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Abstract. This paper develops a simple two-periods model where a seller sell differentiated service goods to two buyers who are connected with each other, where the seller's ability to provide a high quality good is directly dependent on previous successes. I find that in equilibrium, the seller invests more in the buyer who can influence the other buyer. Further, the overall level of investment is highly dependant on the level of complementarity that exists between successes. Finally, in some instances, *ex-ante* welfare is higher in a social network that is not fully connected than a social network that is fully connected.

5520 Words[1]

Contents

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1. Introduction

On February 15th, 2016, Twitter user "joshholzz" tweeted a short video of him uttering "Damn Daniel" while pointing the camera at his friend Daniel. The video became an instant sensation. It reached millions of viewers overnight. In one scene of this video, joshholzz uttered: "Damn Daniel, back at it again with the white Vans", drawing attention to the pair of white Vans shoes that Daniel was wearing.

After being made aware of the video, Vans announced that they will provide Daniel with a life time supply of the now infamous Vans' white shoes. Vans' sales in the following quarter exploded: direct to consumer sales increased by 20% and online sales went up by 30%. Steven Rendle, the President and COO remarked that joshholzz video was one of the driving factors for such an increase [2]. This rise in sales happened despite Daniel's lack of prominence before the video.

Stories such as this one are common in today's social network age, where influential bloggers, "Youtubers", "Instagrammers", and others play an instrumental role in promoting products. This phenomenon is not entirely new. We have repeatedly relied on expert opinion when choosing our consumption goods: a food critic's opinion on a certain restaurant to choose where we dine in, a recommendation from our friends to choose where to get our hair done, a doctor's recommendation on health products to use.

Service providers such as Vans understands the importance of having influential individuals use and endorse their products. As a result, companies see immense value in increasing the odds that these influencers will endorse their products. We know that Vans will not provide all those who buy their white shoes with an unlimited one year supply of the same pair of shoes. So why is it that we still buy them based on Daniel's endorsement? Why is it that Daniel's endorsement still has value even when other consumers know that they are not going to get the same level of service and attention from consuming the same good? And more specifically, how does the seller take advantage of such dynamics in their investment decisions?

This paper explores such dynamics, examining how sellers invest heterogeneously in buyers when they can provide differentiated goods. I approach the analysis using a Career

[2] http://uk.businessinsider.com/damn-daniel-white-vans-sales-vf-corp-2016-4?r=US&IR=T Accessed 10th January, 2017.

Concerns framework where the seller increases their expected revenue by improving their reputation. In this instance, seller reputation is denoted as the posterior probability that the service provided will be of high quality, which is directly linked to learning obtained from providing a high quality good in the previous period. By serving high quality products, seller learns how to serve high quality products to buyers in the future. They then use this learning, in addition to current period investment, to affect current quality.

This paper contributes to the literature in two ways. Firstly, I argue that a buyer only treats high quality realization from other buyers favourably when they know that seller learning will translate to servicing them well in the future. That is, sellers can apply learnings gained from serving B successfully in serving A. This understanding is important as without this feature, heterogeneous investments will simplify to a one seller, one buyer framework.

Secondly, I find that when there is an imbalance in the observation of past history, the seller invests less effort in buyers that observe more market outcome and more in buyers that do not observe others' quality realizations. This stems largely from the rational behaviour of the buyer that does not observe all quality histories. This buyer forms an expectation on the investment level made by the seller to the other buyer. This belief is constant, regardless of whether the actual service provided is of low or high quality, which implies that the seller can *ex-ante* take advantage of this belief to extract revenue from this buyer. Conversely, the seller invests in influential buyers whose quality realization is observable to others in order to extract revenue from other buyers who will treat good news coming from influential buyers favourably.

This paper proceeds as follows. Section 2 surveys the current literature and recent development in understanding seller reputations. Section 3 construct the main model and analyzes its equilibrium behaviours. Section 4 concludes.

2. Literature

Current literature on Seller Reputation originates from Career Concerns models, with the canonical models developed in Holmstrom and Ricart I Costa (1986) and Holmstrom (1999). I rely heavily on this literature in this paper. Career Concerns literature explores a multi-period principal-agent framework where the agent can increase their pay from the principal through improving their reputation. It explores situations where either the principal's action or the outcome is not contractible, giving rise to the power of reputation in inducing the manager to take actions that increases their future payoffs.

Career Concerns was not, however, initially used to model the seller's investment behaviours. Early models exploring seller investment, entry, and exit behaviour focused on the seller finding out about their own type (Jovanovic, 1982), or by forming an expectation over the future levels of competition in Ericson and Makes (1995).

More recently, Career Concerns framework has been used extensively to understand seller reputations. Bar-Isaac and Tadelis (2008) and Cabral (2005) provides a good review of Career Concerns models in understanding seller reputations. Further, recent treatments have focused exclusively on seller investment behaviours by assuming monopolistic behaviour by the seller (Bar-Isaac, 2003). Empirical studies have also been conducted on the real observed behaviour in both sellers and buyers in light of explicit reputation mechanisms (Cabral & Hortacsu, 2010).

These early studies have always examined the role of reputation in exogenous type settings. Some treatments of the reputation effects examine time-variant endogenous types (Board & Meyer-Ter-Vehn, 2013). Board and Meyer-Ter-Vehn (2013) looks at situations where the seller can invest in the quality of their goods in anticipation of a technological breakthrough which happens in accordance with a Poisson process conditional on seller investment. They find a unique work-shirk equilibrium where the seller will shirk on its investment once a desired reputation level is reached and will work if their current reputation is above a certain threshold (if their reputation is still salvageable.) This gives rise to implications surrounding investment decisions made by sellers and consumers as a combined effort to endogenously affect their types and build up their good reputation. Bar-Isaac and Deb (2014) examines the situation where the seller takes multiple action to service different clients.

However, this line of inquiry has so far been limited to a homogeneous good. This paper extends the analysis to heterogenous goods where the seller can distinguish between the investment levels. In fact, this may be more common than conventionally thought. Service oriented sellers providing experience goods always try to keep their formula, so called "industry secrets", away from being publicly disclosed, even amongst clients in fear that their competitor can mirror what they are doing. As a result, such a framework is a common occurrence in any moral hazard type situation.

In these settings, sellers may engage in this strategic investment strategy given their limited resources as a method to build their reputation. For example, given 2 management consultants, one senior and one junior, if the seller is faced with 2 clients, one that has a much higher ability to build up a seller's reputation (having a wider network, being able to send a more credible or trustworthy signal about the consultancy's value), intuition suggests that the firm will assign the senior consultant to the "better" client and the junior consultant to the smaller client.

Liu (2011) explores this information diffusion issue by modelling situations where acquiring information is costly for consumers and shows that dynamics where the seller can exploit its reputation or "milk" their reputation for profit. In this paper however, it is assumed that the firm only services one buyer in one period, not allowing the model to consider social network dynamics that this paper aims to create a framework to explore.

Network effects in consumer actions have been studied extensively. One well known example is the "word-of-mouth" effect (Rob & Fishman, 2005), somewhat similar to the idea pursued in this paper. Rob and Fishman (2005) models a network of connected buyers and focuses on the speed at which a seller's reputation permeates through the network, and correspondingly the seller's investment decisions. Lippert and Spagnolo (2010) extends this idea by explicitly analyzing network strength and word of mouth communications. Galeotti and Sanjeev (2009) examines optimal influence strategy in networks though the focus is on designing marketing strategies as opposed to explicit modelling of investments in quality of goods or services provided to consumers in a network. Bala and Sanjeev (1998), Ballester, Calvo-Armengol, and Zenou (2006) and many others study situations when a network of consumers makes collective decisions as a result of communication between them.

In sum, much research has been done on how sellers accumulate their reputation, focusing exclusively either on endogenous types, hidden actions, or network effects. Little research has explored what happens when sellers can provide differentiated goods to multiple buyers in one period. The following section characterizes a simple model to describe such behaviours.

3. Main Model

In this section, I set up the simplest possible model that can examine reputation-investment dynamics. 1 seller and 2 buyers engage in a 2 periods trade. Two buyers are identical except for their ability to observe other buyer's past. There is an I(fluencer) type and a C(onsumer) type. In period 2, type I can only observe their own realized quality in period 1, while type C can observe the realized quality of both type I and type C in period 1. Buyers have the same non-satiated preferences.

The seller is a risk-neutral expected profit maximizer that produces one unit of product for each buyer in each period. The product can either be of high quality $\bar{q} = 1$ or low quality $\underline{q} = 0$. A buyer type i's product quality at time $t \in \{1, 2\}$ is denoted by q_t^i. The probability that the product's quality is high for buyer i depends on the seller's effort in that period, and the realization of quality for buyer i as well as for buyer $j \neq i$. As a result, the seller chooses 4 levels of effort: $\{e_1^I, e_2^I, e_1^C, e_2^C\}$. The cost of exerting effort is $C_t = C(e_t^C, e_t^I)$. The discount factor for the seller is 1.

We now describe the seller's production technology. The probability that the seller produces a high quality good for buyer i at time t is: $P(q_t^i = 1 | e_t^i, q_{t-1}^i, q_{t-1}^j) = p_t^i(e_t^i, q_{t-1}^i, q_{t-1}^j)$. For simplicity, assume that $e_t^i \in [0, 1]$, $t \in \{0, 1\}$. The effect of past successes on current probability can be thought of as a learning effect. Two learning effects are distinguished here. The first learning effect is the learning that the seller can only use to serve a specific buyer. I will refer to this learning effect as the taste learning effect. The seller acquires taste learning for a buyer by serving that buyer well. The second learning effect is the learning that the seller can use to serve any buyer. I will refer to this learning effect as the skill learning effect. The seller acquires skill learning for a buyer by serving any buyer well. As such, the total learning effect for serving a buyer successfully incorporates both the taste learning for that specific buyer and the skill learning effect. That is, $p_t^i(e, a, b) \geq p_t^i(e, b, a)\ \forall\ 1 \geq a \geq b \geq 0$.

Both $p_t^i(\cdot), C_t(\cdot)$ are twice-differentiable functions that are continuous in e_t^i. Assume $p_t^i(\cdot)$ is increasing in all of its arguments and concave (diminishing returns) in the level of effort. For $C_t(\cdot)$, assume additive separability in effort levels and identical individual cost functions. Further, assume monotonicity and strict convexity (increasing costs).

$$
\begin{gathered}
p_t^C = p_t^I = p_t^i = p_t^i(e_t^i, q_{t-1}^i, q_{t-1}^j) : [0,1]^3 \to [0,1] \\
p_t^i(0,0,0) = 0,\ p_t^i(1,1,1) = 1 \\
\frac{\partial p_t^i}{\partial e_t^i} > 0,\ \frac{\partial^2 p_t^i}{\partial e_t^{i2}} \leq 0 \\
\frac{\partial p_t^i}{\partial q_{t-1}^i} > 0, \frac{\partial p_t^i}{\partial q_{t-1}^j} > 0
\end{gathered} \tag{1}
$$

$$
\begin{gathered}
C_t = C(e_t^C, e_t^I) : [0,1]^2 \to \mathbb{R} \\
\frac{\partial C_t}{\partial e_t^C} > 0,\ \frac{\partial^2 C_t}{\partial e_t^{C2}} > 0 \\
\frac{\partial C_t}{\partial e_t^I} > 0,\ \frac{\partial^2 C_t}{\partial e_t^{I2}} > 0
\end{gathered} \tag{2}
$$

Given that there are only two periods, I will drop the time and individual subscripts on the probability function. When referring to first period probabilities, I will use $p(e^I)$, $p(e^C)$ as there are no quality realizations from the past in period 1. When referring to second period probabilities, I will use $P(q_I, q_C)$, $P(q_C, q_I)$ as it will be evident shortly that there are no incentives for the seller to choose a positive effort in period 2. This simplification is for clarity. When we extend this model to T periods, such simplification may not be possible. Further, assume complementarity in past successes; that is, $P(a,b) > P(0,b) + P(a,0)\ \forall a,b > 0$.

At this point, we ought to stop and discuss the meaning of the complementarity assumption. Complementarity here can be understood as the full utilization of the skill learning gained from one buyer to serving another buyer. In the restaurant example, the chef learns technical skills when serving Diner B successfully. However, they can't fully use this technical knowledge to serve Diner A without knowing A's tastes. The chef can only learn A's taste profile by serving A successfully in the past period. In other words, $P(1,0) > P(1,1) - P(1,0)$

Now, once the seller serves a specific buyer a high quality good, they improve their own skills and learn that buyer's tastes. As a result, the seller can fully utilize the skills they gained to serve that specific buyer in the future. In other words, complementarity only helps the seller utilize the skills they gained from serving other buyers. This implies that as complementarity grows, the learning effect in serving buyer A from serving buyer B successfully, conditional on having served buyer A a low quality good must decrease. The learning effect for buyer A from serving buyer A successfully is not affected as complementarity grows. This distinction will be important when analyzing model dynamics.

The timing of the game is as follows:

Period 0.5: Buyers pay the first period price to the seller.

Period 1: Seller chooses effort level. Quality for period 1 is realized.

Period 1.5: Quality from the first period is observed. Buyers pay the second period price to the seller.

Period 2: Seller chooses effort level for period 2. Quality for period 2 is realized.

Following Holmstrom (1999), and Board and Meyer-Ter-Vehn (2013), I do not model buyer behaviour explicitly in this game. Buyers will always pay the expected quality of the product. Intuitively, we can justify such behaviour by appealing to the fact that the seller is a monopoly, allowing them to extract all surplus from the buyers. Notice that in the second period, the seller has no incentive to exert effort as the effort is chosen after the buyer has paid the price for the second period. As a result, $e_2^{C*} = e_2^{I*} = 0$. Knowing this, in the second period, type C will pay the expected quality given both players' realization of quality in the first period. Type I will pay the expected quality given their own realization of quality in the first period and the correct belief they hold of the realized quality of type C in the first period μ_0.

In period 2, there are 4 possible states. Given the reasoning in the previous paragraph, the total price paid at the beginning of the second period for each of the 4 possible states is: $(P(1,1) + P(1,\mu_0), P(0,1) + P(1,\mu_0), P(1,0) + P(0,\mu_0), P(0,0) + P(0,\mu_0)) \equiv (q_{HH}, q_{HL}, q_{LH}, q_{LL})$. This describes buyers' behaviours fully. Focus now on the seller's behaviour. We normalize prices paid in the first period to 0 and formulate the seller's objective function as a function of effort:

$$
\begin{aligned}
\pi = {} & q_{HH}p(e^C)p(e^I) \\
& + q_{HL}(1 - p(e^C))p(e^I) \\
& + q_{LH}p(e^C)(1 - p(e^I)) \\
& + q_{LL}(1 - p(e^C))(1 - p(e^I)) \\
& - C(e^C, e^I)
\end{aligned} \tag{3}
$$

Now, the first order conditions of this problem reduce to:

$$
\begin{aligned}
p'(e^C)(p(e^I)(q_{HH} - q_{HL}) + (1 - p(e^I))(q_{LH} - q_{LL})) = C_C(e^C, e^I) \\
p'(e^I)(p(e^C)(q_{HH} - q_{LH}) + (1 - p(e^C))(q_{HL} - q_{LL})) = C_I(e^C, e^I)
\end{aligned} \tag{4}
$$

The first order conditions are a standard set of $MR = MC$ equations. On the left hand, the terms in the parenthesis show the effect of a buyer of specific type moving from an unsuccessful investment to a successful investment, multiplied by the change in probability an increase in effort would bring (marginal revenue.) On the righthand is the marginal cost of increasing effort for that type of buyer.

For this game, I characterize a Bayesian-Nash equilibrium, where the buyer's belief on previous level of efforts affects equilibrium behaviour.

3.1. **Full Observation Outcome.** Before analyzing this model with incomplete information, I first solve this model assuming that both buyers can observe each other's history of purchase. This implies that the revenue the seller can extract in the second period is:

$$
\begin{cases}
q_{HH} & = P(1,1) + P(1,1) \\
q_{HL} & = P(0,1) + P(1,0) \\
q_{LH} & = P(1,0) + P(0,1) \\
q_{LL} & = P(0,0) + P(0,0)
\end{cases} \tag{5}
$$

In this case, note that:

$$(6)\qquad \begin{cases} q_{HH} - q_{HL} = P(1,1) + P(1,1) - P(0,1) - P(1,0) = \gamma \\ q_{LH} - q_{LL} = P(1,0) + P(0,1) - P(0,0) - P(0,0) = \theta \\ q_{HH} - q_{LH} = P(1,1) + P(1,1) - P(1,0) - P(0,1) = \gamma \\ q_{HL} - q_{LL} = P(0,1) + P(1,0) - P(0,0) - P(0,0) = \theta \end{cases}$$

The set of first order conditions (4) reduces to:

$$(7)\qquad \begin{aligned} p'(e^C)(p(e^I)\gamma + (1 - p(e^I))\theta) &= C_C(e^C, e^I) \\ p'(e^I)(p(e^C)\gamma + (1 - p(e^C))\theta) &= C_I(e^C, e^I) \end{aligned}$$

As before, these conditions equate the marginal revenue to marginal cost for each buyer. Let $2\omega = \gamma - \theta = q_{HH} - q_{LH} - q_{HL} + q_{LL} = 2(P(1,1) - P(1,0) - P(0,1) - P(0,0))$. ω is the level of complementarity when the realized quality for both buyers is high. Using this, we have the following condition that must be satisfied in equilibrium. We then use this condition to show equilibrium behaviour:

$$(8)\qquad 2\omega(p(e^{C*}) - p(e^{I*})) = \frac{C_I(e^{C*}, e^{I*})}{p'(e^{I*})} - \frac{C_C(e^{C*}, e^{I*})}{p'(e^{C*})}$$

Proposition 1. *In the game with full observations,* $e^{C*} = e^{I*}$

Proof. We prove this proposition by contradiction. Assume that $e^{C*} > e^{I*}$. Given the first order conditions, we have:

$$(9)\qquad \begin{aligned} p(e^{C*}) &> p(e^{I*}) \\ C_C(e^{C*}, e^{I*}) &> C_I(e^{C*}, e^{I*}) \\ p'(e^{C*}) &\leq p'(e^{I*}) \end{aligned}$$

Therefore:

$$(10)\qquad \begin{aligned} p(e^{C*}) - p(e^{I*}) &> 0 \\ \frac{C_I(e^{C*}, e^{I*})}{p'(e^{I*})} - \frac{C_C(e^{C*}, e^{I*})}{p'(e^{C*})} &< 0 \end{aligned}$$

By assumption, $\omega > 0$. As such, the LHS of (8)≥ 0 and RHS of (8)< 0, creating a contradiction. The case with $e^{C*} < e^{I*}$ follows similarly. As a result, $e^{C*} = e^{I*}$ □

This result should not come as a surprise given the symmetric nature of buyers and the information set. We denote the level attained in this equilibrium by $e_f^C = e_f^I = e^*$.

Before continuing, we discuss how equilibrium effort changes as the level of complementarity increases. Rewriting the FOCs using the cost function's additive separability $C(e^C, e^I) = \psi(e^C) + \psi(e^I)$:

$$\begin{aligned} 2p(e^C)\omega + P(1,0) + P(0,1) &= \frac{\psi'(e^I)}{p'(e^I)} \\ 2p(e^I)\omega + P(1,0) + P(0,1) &= \frac{\psi'(e^C)}{p'(e^C)} \end{aligned} \tag{11}$$

Notice $P(1,0) + P(0,1) = P(1,1) - \omega$. Then, using the fact that in equilibrium, $e_f^C = e_f^I = e^*$:

$$2p(e^*)\omega + P(1,1) - \omega = \frac{\psi'(e^*)}{p'(e^*)} \tag{12}$$

Totally differentiating with respect to ω, we will have:

$$\begin{aligned} 2p(e^*) + 2p'(e^*)\frac{de^*}{d\omega}\omega - 1 &= \frac{\psi''(e^*)p'(e^*) - \psi'(e^*)p''(e^*)}{p'(e^*)^2}\frac{de^*}{d\omega} \\ \Rightarrow \frac{de^*}{d\omega} &= \frac{p'(e^*)^2\Big[1 - 2p(e^*)\Big]}{2p'(e^*)^3\omega - \psi''(e^*)p'(e^*) + \psi'(e^*)p''(e^*)} \end{aligned} \tag{13}$$

If $2p(e^*) < 1$ and $2p'(e^*) < \dfrac{\psi''(e^*)p'(e^*) - \psi'(e^*)p''(e^*)}{p'(e^*)^2}$, the derivative is negative - equilibrium effort goes down with increased complementarity. The second inequality imposes a lower bound on the rate of increase of the ratio between marginal cost and marginal product of effort. The overall decrease in effort is due to the decrease in value of only serving one buyer successfully and an increased value in serving both buyers successfully. Notice that as long as the first condition is satisfied, equilibrium effort will at the least go down when complementarity is marginally increased from $\omega = 0$.

For example, in the chef example used above, by exerting more effort, the chef is increasing the likelihood of fully utilizing techniques learned from serving one diner to serving the other diner. However, if the overall learning effect for second period success is sufficiently larger than the marginal product of exerting effort, and if the rate at which cost increases is much higher than the marginal product in increasing effort, the chef may want to invest less effort when complementarity is higher.

3.2. **No Observation Outcome.** Secondly, I analyze the game when neither buyers observe each other's realization of quality. Denoting type C's belief about the effort level for type I as μ_I and type I's belief about the effort level for type C as μ_C:

$$\begin{cases} q_{HH} &= P(1,\mu_I) + P(1,\mu_C) \\ q_{HL} &= P(0,\mu_I) + P(1,\mu_C) \\ q_{LH} &= P(1,\mu_I) + P(0,\mu_C) \\ q_{LL} &= P(0,\mu_I) + P(0,\mu_C) \end{cases} \tag{14}$$

Correspondingly,

$$\begin{cases} q_{HH} - q_{HL} = P(1,\mu_I) - P(0,\mu_I) \\ q_{LH} - q_{LL} = P(1,\mu_I) - P(0,\mu_I) \\ q_{HH} - q_{LH} = P(1,\mu_C) - P(0,\mu_C) \\ q_{HL} - q_{LL} = P(1,\mu_C) - P(0,\mu_C) \end{cases} \tag{15}$$

The first order conditions of this problem reduce to:

$$\begin{aligned} P(1,\mu_I) - P(0,\mu_I) &= \frac{C_C(e^C, e^I)}{p'(e^C)} \\ P(1,\mu_C) - P(0,\mu_C) &= \frac{C_I(e^C, e^I)}{p'(e^I)} \end{aligned} \tag{16}$$

where $\mu_I = p(e^I)$ and $\mu_C = p(e^C)$ in equilibrium. It is easy to see that one solution is $e^I = e^C$. We denote the level attained in this equilibrium by e_n^C, e_n^I. A natural question that arises is whether the seller invests more in this scenario than in the previous scenario. It suffices, given the symmetric nature of the equilibrium, for us to compare the effort level for one type of buyer.

We see that the answer depends on the level of complementarity. To explore the idea, impose a further restriction on the degree of complementarity: $P(1,a) - P(0,a) - P(1,0) <$

$P(0,1)\forall a \in [0,1]$. This assumption means that the complementarity must be sufficiently small. Given this, we will have:

$$P(1,\mu_I) - P(0,\mu_I) < P(0,1) + P(1,0) \tag{17}$$

Which then directly implies that the investment level in no observation case will be lower than the level afforded in the full observation case. Intuitively, if the complementarity is sufficiently weak, the seller's incentive to invest in each buyer will exceed the seller's disincentive of investment from the buyers not observing actual quality realization from the other buyer.

Conversely, if the degree of complementarity is sufficiently large, $P(1,a) - P(0,a) - P(1,0) \geq P(0,1)\forall a \in [0,1]$, the seller's effort will be greater than or equal to the full observation scenario for both buyers as the return on higher level of effort exerted will be higher. However, as $P(1,0) - P(0,0) - P(1,0) = 0 = P(0,1)$, no skill learning is transferable. Intuitively, seller invests more in this case as they can take more advantage of the complementarity as buyer's belief on the level of complementarity does not depend on actual realization of quality, but to the effort invested. For intermediate values, the result is ambiguous and depends on the specific functional forms.

3.3. **Limited Observation Outcome.** Finally, we analyze the equilibrium outcome in the case of limited observations where type C buyer observes past qualities for type I while type I can only observe their own history of quality. We have:

$$\begin{cases} q_{HH} - q_{HL} = P(1,1) - P(0,1) \\ q_{LH} - q_{LL} = P(1,0) - P(0,0) \\ q_{HH} - q_{LH} = P(1,1) - P(1,0) + P(1,\mu_0) - P(0,\mu_0) \\ q_{HL} - q_{LL} = P(0,1) - P(0,0) + P(1,\mu_0) - P(0,\mu_0) \end{cases} \tag{18}$$

This implies that the belief μ_0 held by type I is only relevant when we consider revenue differences between quality realizations of type I, holding the quality realization of type

C constant. Reduce the first order conditions to:

$$(19)\qquad \begin{aligned} p(e^I)(q_{HH}-q_{HL}-q_{LH}+q_{LL})+q_{LH}-q_{LL} &= \frac{C_C(e^C,e^I)}{p'(e^C)} \\ p(e^C)(q_{HH}-q_{LH}-q_{HL}+q_{LL})+q_{HL}-q_{LL} &= \frac{C_I(e^C,e^I)}{p'(e^I)} \end{aligned}$$

Notice that $q_{HH}-q_{LH}-q_{HL}+q_{LL}=\omega$, as defined in the full observation case. Solving this system implies the following condition:

$$(20)\qquad \frac{C_I(e^C,e^I)}{p'(e^I)}-\frac{C_C(e^C,e^I)}{p'(e^C)}=q_{HL}-q_{LH}+\omega(p(e^{C*})-p(e^{I*}))$$

Using this condition, and denoting equilibrium effort level in this case by e^C_l, e^I_l, I can prove the following proposition regarding equilibrium behaviour:

Proposition 2. *In equilibrium,* $e^C_l < e^I_l$

Proof. I prove this by contradiction. Assume that $e^{C*} \geq e^{I*}$. As before, this implies that:

$$(21)\qquad \begin{aligned} \frac{C_I(e^C,e^I)}{p'(e^I)}-\frac{C_C(e^C,e^I)}{p'^C(e^C)} &\leq 0 \\ p(e^{C*})-p(e^{I*}) &\geq 0 \end{aligned}$$

Now, evaluate $q_{HL}-q_{LH}$:

$$(22)\qquad q_{HL}-q_{LH}=P(0,1)+P(1,\mu_0)-P(1,0)-P(0,\mu_0)$$

$$(23)\qquad \begin{aligned} P(0,1)-P(0,\mu_0) &\geq 0 \\ P(1,\mu_0)-P(1,0) &> 0 \end{aligned}$$

Given the symmetry of the probability function and assuming an interior solution where $\mu_0 > 0$. Therefore, $q_{HL}-q_{LH} > 0$. We also know that $\omega > 0$. This creates the desired contradiction. As a result, $e^C_l < e^I_l$.

□

This proposition shows that in equilibrium, the seller will prioritize investing in type I buyer, whose realization of quality can be observed by type C buyer. This stems from the fact that an unsuccessful investment for type C will only reduce the price paid in the next period by type C consumer while an unsuccessful investment for type I will reduce the price paid by both C and I. The size of this conflict is $q_{HL} - q_{LH}$, or the difference in revenue when only type I observes a successful investment, and when only type C observes a successful investment.

Intuitively, as the size of this conflict decreases, so does the difference in effort levels between the two buyers. The conflict's size depends on the difference between the marginal effect of a successful investment in the previous period for both buyers to the marginal effect of a successful investment for one particular buyer in the previous period to current period's quality. This is central to this model - if type C buyer believes that a successful outcome observed by type I has little or no bearing to their own realization of quality, they will ignore such information and diminishes the seller's incentives to invest in type I. As such, the seller will have more incentive to invest more in type C buyer in the first period to affect their pay in the second period.

This idea is illustrated in the following example. Cosmetic companies regularly solicit celebrities to endorse their beauty products. Regular consumers understand that celebrities who endorsed such products receive significant kick back from the companies. The consumers thus expect to receive worse services from the seller. However, consumers also expect that the cosmetic product must have met some quality standard in order for the celebrity in question to use it. As long as there are skill learning effects, the network reputation effect still exists.

3.4. **Comparison: Full Observation and Limited Observation.** Having analyzed the model in its three versions, I proceed by comparing the two most important iterations of this model: full observations and limited observations. Equilibrium effort as well as surplus are compared here. As stated previously, in the efficient case, the symmetric optimal effort level is characterized by:

$$\begin{aligned} 2p(e_f^I)\omega + P(1,0) + P(0,1) &= \frac{C_C(e_f^C, e_f^I)}{p'(e_f^C)} \\ 2p(e_f^C)\omega + P(1,0) + P(0,1) &= \frac{C_I(e_f^C, e_f^I)}{p'(e_f^I)} \end{aligned} \tag{24}$$

In the imperfect observation case, the optimal effort levels are characterized by:

$$p(e_l^I)\omega + P(1,0) = \frac{C_C(e_l^C, e_l^I)}{p'(e_l^C)}$$
$$p(e_l^C)\omega + P(0,1) + P(1,p(e_l^C)) - P(0,p(e_l^C)) = \frac{C_I(e_l^C, e_l^I)}{p'(e_l^I)} \tag{25}$$

Comparing equilibrium efforts in the general case is complicated. As a result, I use the following functional form:

$$p(e) = ae$$
$$P(q_i, q_j) = bq_i + cq_j + dq_iq_j$$
$$a, b, c, d \in [0,1]$$
$$a + b + c + d = 1 \tag{26}$$
$$b \geq c + d$$
$$C(e^C, e^I) = \frac{1}{2}e^{C2} + \frac{1}{2}e^{I2}$$

Here, the degree of complementarity is measured by d, c denotes skill learning the seller can use directly, and b encodes both skill and taste learning effects. Using this functional form, the first order conditions for the full observation will be:

$$2ade^I + b + c = \frac{e^C}{a}$$
$$2ade^C + b + c = \frac{e^I}{a} \tag{27}$$

And that of the limited observation case will be:

$$ade^I + b = \frac{e^C}{a}$$
$$2ade^C + b + c = \frac{e^I}{a} \tag{28}$$

The equilibrium effort level in the full observation case will be:

$$e_f^C = e_f^I = e^* = \frac{a(b+c)}{1-2da^2} \tag{29}$$

Correspondingly, the equilibrium effort levels in the limited observation case will be characterized by:

$$\begin{aligned} e_l^I &= \frac{a(2a^2bd + b + c)}{1 - 2d^2a^4} \\ e_l^C &= a(ade_l^I + b) \end{aligned} \tag{30}$$

I now show that for any positive level of complementarity, $e_l^I < e^*$.

Proposition 3. $e_l^I < e^*$

Proof. Showing this is straightforward. We simply take the difference between the equilibrium effort levels:

$$\begin{aligned} e_l^I - e^* &= \frac{a(2a^2bd + b + c)}{1 - 2d^2a^4} - \frac{a(b + c)}{1 - 2da^2} \\ &= \frac{(1 - 2da^2)a(2a^2bd + b + c) - (1 - 2d^2a^4)(ba + ca)}{(1 - 2d^2a^4)(1 - 2da^2)} \\ &= \frac{2da^3(a^2dc - a^2db - c)}{(1 - 2d^2a^4)(1 - 2da^2)} \end{aligned} \tag{31}$$

The denominator is always positive (proof in the appendix.) Using the fact that $b \geq c$, the numerator will be non-positive. As a result, the difference is smaller than 0.

□

Intuitively, the return from investing in type C diminished greatly between the full observation case and limited observation case. In the full observation case, successful provision for type C affects both C and I's payment in the second period. However, in the first period, this effect does not exist. On the other hand, as the amount invested in type C diminishes, the level of complementarity type I experiences will similarly diminish. As a result, the incentive for the seller to exert effort in type I also diminishes.

This analysis illustrates an important dynamic of reputation. When information asymmetry exists, expectations on another buyer's quality is still important to a buyer who doesn't observe others' realization of quality. However, this expectation is imperfect, diminishing the seller's incentive to take advantage of such effects, leading to our result.

Finally, I analyze the *ex-ante* surplus under the full observation and limited observation case. In our set up, the seller captures all of the surplus. Thus, it suffices to look at the expected profit attained by the seller. As the level of effort decreases for both types, one might suspect that ex-ante surplus goes down in the limited observation case. I illustrate here that this need not be the case. I first proceed by analyzing the difference when there is no complementarity, which simplifies our analysis considerably. I then provide intuitions for when there is a positive level of complementarity.

Without complementarity, $P(a,b) = P(a,0) + P(0,b)$, the seller can fully utilize the skill learning from serving buyer B successfully in serving buyer A without knowing A's tastes. Notice that the conditions for the fully connected case (denoted f) and limited observation case (denoted l) will reduce to:

$$\begin{aligned} P(1,0) + P(0,1) &= \frac{C_C(e_f^*, e_f^*)}{p'(e_f^*)} \\ P(1,0) + P(0,1) &= \frac{C_I(e_f^*, e_f^*)}{p'(e_f^*)} \end{aligned} \tag{32}$$

$$\begin{aligned} P(1,0) &= \frac{C_C(e_l^C, e_l^I)}{p'(e_l^C)} \\ P(0,1) + P(1,0) &= \frac{C_I(e_l^C, e_l^I)}{p'(e^I)} \end{aligned} \tag{33}$$

Which implies that $e_l^C < e_l^I = e_f^*$; that is, effort invested in type I is the same as the fully connected case with type C's investment being lower. The difference between the profit levels is:

$$\pi_f - \pi_l = P(1,0)(p(e^*) - p(e^C)) + P(0,1)p(e^*) - P(0, p(e^C)) - C(p(e^*), p(e^*)) + C(p(e^C), p(e^*)) \tag{34}$$

The full algebraic operations are left in the appendix. Notice that this difference is smaller than 0 when:

$$P(1,0)(p(e^*) - p(e^C)) + P(0,1)p(e^*) - P(0, p(e^C)) < C(p(e^*), p(e^*)) - C(p(e^C), p(e^*)) \tag{35}$$

In other words, it is possible for the ex-ante profit level reached in the limited observation case to be higher than the ex-ante profit level reached in the full observation case. For example, if the second period probability function is homogeneous of order 1, we will have $P(1,0)(p(e^*) - p(e^C)) > P(0,1)p(e^*) - P(0,1)p(e^C) = P(0,1)p(e^*) - P(0,p(e^C))$, as $P(1,0) \geq P(0,1)$. As a result, this condition is more likely to hold for higher levels of skills learning as compared to taste learning. Higher skill learning effect encourages the seller to invest more in type I buyers in the limited observation case as good news is treated more favourably by type C buyers. This then also raises revenues extracted from type I. However, realize that in general, equilibrium effort goes down for both buyers. As such, this welfare improvement comes from lower cost associated with effort, as opposed to producing better products.

To see what happens for higher levels of complementarity, notice that:

$$\begin{cases} q^E_{HH} - q^{NE}_{HH} = P(1,1) - P(1,\mu_0) \geq 0 \\ q^E_{HL} - q^{NE}_{HL} = P(1,0) - P(1,\mu_0) \leq 0 \\ q^E_{LH} - q^{NE}_{LH} = P(0,1) - P(0,\mu_0) \geq 0 \\ q^E_{LL} - q^{NE}_{LL} = -P(0,\mu_0) \leq 0 \end{cases} \tag{36}$$

As the level of complementarity increases, we expect both effort levels to decrease. This is due to the higher marginal benefit in a realization of a high quality good. The relative gap between the effort levels for type C and type I will widen (though the absolute gap will be smaller), as type I's ex-ante expected quality also depends on the investment level afforded to type C, not just the actual realization for type C.

As for welfare, the overall effect is ambiguous but I provide an intuition here. For sufficiently high complementarity, ex-ante welfare in the limited observation case will be higher than the full observation case. This is due to a smaller complementarity elasticity of effort for type I. As complementarity increases, effort exerted towards type I doesn't respond largely, which improves the probability of type I experiencing a successful investment and the revenue associated in those cases.

4. Conclusion

This paper establishes a simple yet illustrative framework to analyze seller reputation effects in a social network. We find that given strong enough complementarity, *ex-ante* welfare can be higher when buyers have a limited network as opposed to a fully connected one. The improvement in welfare is positively correlated with the level of complementarity that exists between successful investments for the buyers.

The findings imply that we should treat markets that exhibit different levels of complementarity between past successes differently. Markets where the skill learning effects dominate, and as a result have lower levels of complementarity, such as traditional consumer goods market should have a more connected network. On the other hand, markets where complementarity is high (that is, the effect of taste learning is higher) can reach a higher *ex-ante* welfare by having the seller invests more in an influencer.

This paper also contributes to the literature by providing an intuition behind why buyers might choose to respond to advertisements and celebrity endorsements, knowing that the seller can and will invest less in the regular buyers. The crucial effect lies in the differentiation between the skill learning effects and the taste learning effects.

This finding has several policy implications. Until now, conventional thinking in economics has pointed towards information asymmetry as a source of reduction in overall welfare. However, given the right circumstances, information asymmetry may actually improve overall *ex-ante* welfare. This paper does not give implications surrounding inequality that could be created as a result of this information asymmetry. However, these welfare improvements are largely due to reduction in costs as opposed to an improvement in quality. We can examine potential compensation mechanisms to address such inequality to reach a Pareto improvement from the full observation case.

Future research can examine this welfare dynamics more closely. Further, it can extend the model to examine the cases of having more than 2 buyers, with each buyer being distinguished by a parameter that determines their likelihood of being able to communicate their realization of quality to others in the market. It will also be productive to check whether this results is robust if buyers and sellers interact repeatedly, not limited to just 2 periods.

Finally, this model also informs possible empirical explorations in this area. Researchers may wish to focus on the learning effects, especially the complementarity between successes to examine investment decisions by sellers located in different sectors.

Appendices

A. Manipulation of welfare function without complementarity

$$\begin{aligned}\pi_f =& 2P(1,1)p(e^*)^2 + 2[P(0,1)+P(1,0)]p(e^*)(1-p(e^*))\\ \pi_l =& [P(1,1)+P(1,p(e^C))]p(e^*)p(e^C) + [P(0,1)+P(1,p(e^C)]p(e^*)(1-p(e^C))\\ &+ [P(1,0)+P(0,p(e^C))](1-p(e^*))p(e^C) + (1-p(e^*))(1-p(e^C))P(0,p(e^C))\end{aligned} \tag{37}$$

$$\begin{aligned}A &= 2P(1,1)p(e^*)^2 - [P(1,1)+P(1,p(e^C))]p(e^*)p(e^C)\\ &= p(e^*)[2P(1,1)p(e^*) - [P(1,1)+P(1,p(e^C))]p(e^C)]\\ &= p(e^*)[P(1,1)(p(e^*)-p(e^C)) + P(1,1)p(e^*) - P(1,p(e^C))p(e^C)]\end{aligned} \tag{38}$$

(39)

$$\begin{aligned}B &= [P(0,1)+P(1,0)]p(e^*)(1-p(e^*)) - [P(0,1)+P(1,p(e^C)]p(e^*)(1-p(e^C))\\ &= p(e^*)[[P(0,1)+P(1,0)](1-p(e^*)) - [P(0,1)+P(1,p(e^C))](1-p(e^C))]\\ &= p(e^*)[P(1,0)-P(1,p(e^C)) - P(0,1)(p(e^*)-p(e^C)) - P(1,0)p(e^*) + P(1,p(e^C))p(e^C)]\end{aligned}$$

$$A+B = p(e^*)[P(1,0)(p(e^*)-p(e^C)) - P(0,p(e^C)) + p(e^*)P(0,1)] \tag{40}$$

$$\begin{aligned}C &= [P(0,1)+P(1,0)]p(e^*)(1-p(e^*)) - [P(1,0)+P(0,p(e^C))](1-p(e^*))p(e^C)\\ &= (1-p(e^*))[P(1,0)(p(e^*)-p(e^C)) + P(0,1)p(e^*) - P(0,p(e^C))p(e^C)]\end{aligned} \tag{41}$$

$$D = -(1-p(e^*))(1-p(e^C))P(0,p(e^C)) \tag{42}$$

$$C+D = (1-p(e^*))[P(1,0)(p(e^*)-p(e^C)) + P(0,1)p(e^*) - P(0,p(e^C))] \tag{43}$$

$$\begin{aligned}\pi_f - \pi_l &= A+B+C+D\\ &= P(1,0)(p(e^*)-p(e^C)) + P(0,1)p(e^*) - P(0,p(e^C))\end{aligned} \tag{44}$$

B. Proof that $(1 - 2a^4d^2)(1 - 2a^2d) > 0$

We begin by noting that $a(1-a) \geq ad \geq a^2d \geq a^4d^2$ as $a+b+c+d = 1$. $a(1-a)$ is maximized at $a = \frac{1}{2}$, giving the value of $\frac{1}{4}$. This implies that both equations in the parenthesis will be positive, implying that their product is also positive.

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